

Math Virtual Learning

Precalculus with Trigonometry

Students will simplify expressions using
Fundamental Trig Identities

April 23, 2020



Precalculus with Trigonometry

Lesson: April 23nd, 2020

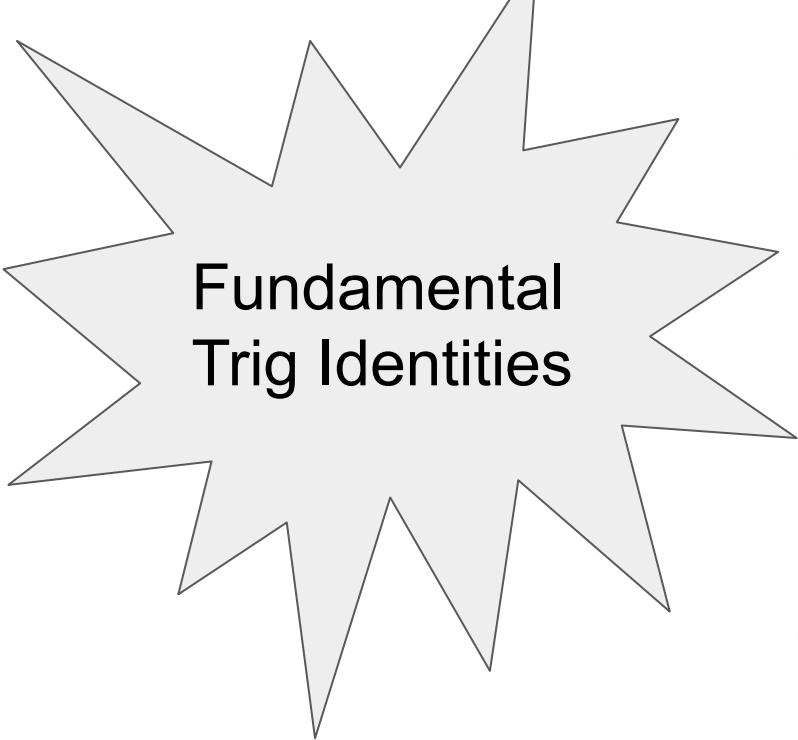
Objective/Learning Target:
Students will simplify expressions using
Fundamental Trig Identities

Watch Video:



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Fundamental Trig Identities

Trigonometric Identities

Reciprocal Identities

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Simplifying Expressions Using Trig Identities

Example 1: Simplify $\tan(x)\cos(x)$

Oftentimes it's easiest to simplify trig expressions by **rewriting each function in terms of sine and cosine**.

By the Quotient Identity, rewrite $\tan(x)$ as $\sin(x)/\cos(x)$, cancel the $\cos(x)$ terms and the expression simplifies to equal $\sin(x)$

$$\tan x \cos x$$

$$\frac{\sin x}{\cos x} \cos x$$

$$\tan x \cos x = \sin x$$

Example 2

Simplify $\sec(x)/\csc(x)$

By the Reciprocal and Quotient Identities, the expression simplifies to $\tan(x)$

$$\begin{aligned} & \frac{\sec x}{\csc x} \\ & \frac{1}{\cos x} \cdot \frac{1}{\frac{1}{\sin x}} \\ & \frac{1}{\cos x} \cdot \sin x \\ & \frac{\sin x}{\cos x} \\ & = \tan x \end{aligned}$$

Example 3: Simplify $\cos \theta + \cos \theta(\tan^2 \theta)$

Notice that the terms in the expression $\cos \theta + \cos \theta(\tan^2 \theta)$ have a common factor of $\cos \theta$, so start by factoring this common term out.

$$\begin{aligned}\cos \theta + \cos \theta(\tan^2 \theta) \\ \cos \theta(1 + \tan^2 \theta)\end{aligned}$$

Now, use the trigonometric identity $1 + \tan^2 \theta = \sec^2 \theta$, substitute, and simplify.

$$\begin{aligned}\cos \theta(1 + \tan^2 \theta) \\ = \cos \theta(\sec^2 \theta) \\ = \cos \theta\left(\frac{1}{\cos^2 \theta}\right) \\ = \frac{1}{\cos \theta} \\ = \sec \theta\end{aligned}$$

Practice

Use the Fundamental Trigonometric Identities to simplify the following expressions.

1. $\frac{\tan \theta}{\cot \theta}$

2. $(\sec^2 \theta - 1) \cos^2 \theta$

3. $\csc \theta - \cos \theta \cot \theta$

4. $\sin^2 x(1 + \cot^2 x)$

Answers to practice

1. $\tan\theta/\cot\theta$

$= \tan\theta/1/\tan\theta$ Reciprocal Identity

$= \tan^2 \theta$

2.

$(\sec^2 \theta - 1)\cos^2 \theta$

$= (\tan^2 \theta)\cos^2 \theta$ Pythagorean Identity

$= \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)\cos^2 \theta$ Quotient Identity

$= \sin^2 \theta$ Multiply and divide out common factor.

3.

$= \frac{1}{\sin \theta} - \cos \theta \left(\frac{\cos \theta}{\sin \theta}\right)$ Reciprocal and Quotient Identities

$= \frac{1 - \cos^2 \theta}{\sin \theta}$ Write as a fraction with a common denominator.

$= \frac{\sin^2 \theta}{\sin \theta}$ Pythagorean Identity

$= \sin \theta$ Divide out common factor of $\sin \theta$.

4.

$= \sin^2 x \csc^2 x$ Pythagorean Identity

$= \sin^2 x \frac{1}{\sin^2 x}$ Reciprocal Identity

$= 1$ Cancel out common factor

Additional Practice and Resources:

Additional Resource Videos:

[Simplifying Using Pythagorean Identities - Khan Academy](#)

[Simplifying Expressions using Fundamental Identities](#)

Additional practice

[Practice Simplifying Trig Expressions](#)

**Answers to practice are on the next slide*

Answer to simplifying trig expressions practice

